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# Dynamics of Competition Between Incumbent and Emerging Network Technologies

Youngmi Jin

*University of Pennsylvania*, [youngmi@seas.upenn.edu](mailto:youngmi@seas.upenn.edu)

Soumya Sen

*University of Pennsylvania*, [soumyas@princeton.edu](mailto:soumyas@princeton.edu)

Roch A. Guérin

*University of Pennsylvania*, [guerin@acm.org](mailto:guerin@acm.org)

kartik Hosanagar

*University of Pennsylvania*, [kartikh@wharton.upenn.edu](mailto:kartikh@wharton.upenn.edu)

Zhi-Li Zhang

*University of Minnesota*

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# Dynamics of Competition Between Incumbent and Emerging Network Technologies

## **Abstract**

The Internet is by all accounts an incredible success, but in spite or maybe because of this success, its deficiencies have come under increasing scrutiny and triggered calls for new architectures to succeed it. Those architectures will, however, face a formidable incumbent in the Internet, and their ability to ultimately replace it is likely to depend equally on technical superiority as on economic factors. The goal of this paper is to start developing models that can help provide a quantitative understanding of a competition between the Internet and a new system, and show what factors affect it most strongly. A model for the adoption of competing network technologies by individual users is formulated and solved. It accounts for both the intrinsic value of each technology and the positive externalities derived from their respective numbers of adopters. Using this model, different configurations are explored and possible outcomes characterized. More importantly, configurations are identified where small differences in the attributes of either technology can lead to vastly different results. The paper provides initial results that can help identify parameters that significantly affect the likelihood of success of new network technologies.

## **Keywords**

Networks, Economy, Technology Diffusion

## **Comments**

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# Dynamics of Competition Between Incumbent and Emerging Network Technologies \*

Youngmi Jin  
ESE, U.Pennsylvania  
youngmi@seas.upenn.edu

Soumya Sen  
ESE, U.Pennsylvania  
ssoumya@seas.upenn.edu

Roch Guérin  
ESE, U.Pennsylvania  
guerin@ee.upenn.edu

Kartik Hosanagar  
Wharton, U.Pennsylvania  
kartikh@wharton.upenn.edu

Zhi-Li Zhang  
CSE, U.Minnesota  
zhzhang@cs.umn.edu

## ABSTRACT

The Internet is by all accounts an incredible success, but in spite or maybe because of this success, its deficiencies have come under increasing scrutiny and triggered calls for new architectures to succeed it. Those architectures will, however, face a formidable incumbent in the Internet, and their ability to ultimately replace it is likely to depend equally on technical superiority as on economic factors. The goal of this paper is to start developing models that can help provide a quantitative understanding of a competition between the Internet and a new system, and show what factors affect it most strongly. A model for the adoption of competing network technologies by individual users is formulated and solved. It accounts for both the intrinsic value of each technology and the positive externalities derived from their respective numbers of adopters. Using this model, different configurations are explored and possible outcomes characterized. More importantly, configurations are identified where small differences in the attributes of either technology can lead to vastly different results. The paper provides initial results that can help identify parameters that significantly affect the likelihood of success of new network technologies.

## Categories and Subject Descriptors

H.1.0 [Information Systems]: Models and Principles—General

## General Terms

Economics, Management

## Keywords

Technology Adoption, Technology Diffusion, User Heterogeneity, Network Externality

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## 1. INTRODUCTION

Advances in technology often see newer and better solutions replacing older ones. Networking is no exception, and the Internet displaced the traditional phone network as the de facto communication infrastructure, for example. And it also displaced more direct competitors, packet data network technologies such as ATM, Frame Relay, SNA and others. But this displacement took a long time. In other words, technical superiority alone is no guarantee for success, especially in the presence of a strong incumbent. A large body of work has indeed been devoted to this issue, but networking technologies exacerbate this phenomenon because of the many factors that affect their value, and in particular the benefits of a large installed base. The Internet itself provides a perfect example of such complex interactions, with its long-standing “migration” from IPv4 to IPv6. In spite of a known need (the eventual exhaustion of IPv4 addresses) and intrinsic technical advantages (security, mobility, etc.), uncertainties regarding technology maturity and stability and difficulties in ensuring a transparent transition for existing services, have stymied all large-scale conversion attempts. Could this have been predicted, and were there steps that could have been taken to remedy the situation and hasten a transition that most if not all view as positive and ultimately necessary? In this paper, we propose a model to study both the dynamics of diffusion of new network technologies in the presence of an incumbent, and their eventual equilibrium adoption levels.

We start with a model of individual user’s utility and build an aggregate diffusion model that is consistent with individual rational decision-making in adopting network technologies. In our model, users can be heterogeneous in the way they value the services deployed on top of any network technology. The drawback of modeling individual level decision-making and user heterogeneity is that it makes the diffusion model complex. However, the cost of this complexity is outweighed by two benefits. First, the model allows us to understand both individual-level and system-level dynamics, and explore how small changes in system parameters can affect individual decisions and ultimately lead to very different system-level outcomes. Second, the model allows us to identify and explain interesting phenomena such as the presence of multiple equilibria and the potential for both network technologies to coexist.

Although preliminary, we have four main findings to report from our initial analysis:

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- In many instances, multiple equilibrium adoption levels may exist for the same set of parameters (i.e., price, quality levels, etc.). The specific equilibrium attained depends on the initial condition for the diffusion process. Stated differently, the equilibrium adoption levels depend on entry timing of the new network technology.
- It is possible for both the incumbent and entrant technologies to coexist in equilibrium, even in the absence of gateways or converters.
- Even though the entrant may seem to be diffusing well, it is often doomed to fail if its growth rate is slower than that of the incumbent. Thus, the observer can be misled by the apparent steady pace of diffusion of a new network technology, and should closely monitor relative growth rates and network benefits.
- When multiple equilibria exist, we observe that small changes in parameters can sometimes have a big impact on the equilibrium adoption levels. Thus, decision makers have to be particularly cautious with pricing, entry timing and other deployment decisions.

The rest of this paper is organized as follows: Section 2 discusses prior work and positions our work in the literature. Section 3 introduces our model and problem formulation. Section 4 characterizes equilibrium adoption levels, while Section 5 provides a numerical study of diffusion dynamics involving two competing network technologies. We discuss the limitations of this study and conclude the paper with remarks on future work in Section 6.

## 2. RELATED WORK

There are two streams of work relevant to our study. The first relates to the literature on adoption of incompatible technologies. The second relates to the literature on new product/technology diffusion. We discuss both below.

**Adoption of incompatible technologies:** Adoption of incompatible technologies in the presence of network externalities has been treated in a number of prior works, e.g., [4, 3]). When technologies are incompatible, users of a technology can only reach other users of the same technology and, as a result, the value a user derives from a technology is a function of the size of its installed base (these are referred to as network benefits or network externalities). The main focus of the literature has been on the impact of converters that help make one technology partially compatible with the other. The key finding of these works is that network externalities can often lead to multiple equilibria and that converters have a significant impact on equilibrium adoption levels. A recent paper by Joseph et al. [7] extends the results in the context of new network architectures. These papers consider static models and do not model the exact convergence path that leads to any equilibrium. As a result, they do not allow the observer to infer how the diffusion process selects one of several equilibria and also make it difficult to devise dynamic policies.

**New Product Diffusion:** Modeling the diffusion of new products and technologies has a long tradition in Marketing [1] and is still an active area of research (see [8] for an overview of this literature). The vast majority of these models focus on aggregate adoption dynamics without explicitly modeling individual decision making processes. The advantage of the approach is that it results in relatively simple

diffusion models that can then be used to study dynamic policies (e.g., dynamic pricing). Unfortunately, these aggregate models do not shed sufficient light on the decision processes that lead to certain system dynamics or the exact mechanism through which various decision variables (pricing, quality, advertising, etc.) impact adoption decisions. A few models have focused on individual-level adoption (for example [2, 6]). These models provide far greater insight into the mechanism through which rational individual decision-making results in aggregate system dynamics. Given the complexity of these models, much of the progress to date has been in settings with one single technology. In contrast, the adoption of new network technologies and architectures is often influenced by the presence of incumbents. In order to fully understand the drivers of their success and failures, it is important to model the role of the incumbent.

Prior work reveals two themes. First, while the literature on adoption of incompatible technologies has modeled network externalities and demonstrated it can result in multiple equilibria, these models are static and do not provide deep insight on the convergence process or path. Two, the diffusion literature has primarily focused on aggregate models and the few individual-level models have been for single technologies diffusing in isolation. Because new network architectures are often deployed in the presence of incumbents, understanding diffusion of new network architectures requires that we model dynamic processes and incorporate the role of incumbents. This is the question we turn to now.

## 3. PROBLEM FORMULATION

This section introduces our model for studying adoption dynamics between two competing network technologies.

### 3.1 User's Decision Process

Consider two network technologies, labeled 1 and 2, representing the incumbent and entrant respectively: for example, IPv4 versus IPv6 or the current Internet versus a clean-slate alternative. The quality of technology  $i$  is denoted by  $q_i > 0$  and its price  $p_i > 0$ , for  $i = 1, 2$ . We assume a fixed population of  $N$  users, with  $N_i$  the number of users adopting technology  $i = 1, 2$ . The proportion of users who adopt technology  $i$  is denoted by  $x_i = \frac{N_i}{N}$ , with  $\underline{x} = (x_1, x_2) \in S$ , where

$$S = \{(x_1, x_2) \mid x_1 + x_2 \leq 1, 0 \leq x_1 \leq 1, 0 \leq x_2 \leq 1\}$$

denotes the set of possible adoption levels. An end user's surplus from technology  $i$  is modeled as

$$U_i(\theta, \underline{x}) = \theta q_i + v(x_i) - p_i \quad (1)$$

$\theta q_i$  is the standalone benefit that the user obtains from technology  $i$ , with  $\theta \in [0, 1]$  being an individual preference parameter that reflects how much the user values the technology. The value of  $\theta$  varies across users capturing their heterogeneity. Individual values of  $\theta$  are private, but their distribution, denoted by  $F(\theta)$ , is known.  $v(x_i) \geq 0$  denotes the positive network externalities or benefits that the user derives from other adopters of technology  $i$ . These benefits increase with the number of adopters, so that  $v(x_i)$  is a nondecreasing function with  $v(0) = 0$ . There exist several techniques to estimate the user-specific weights in utility functions (e.g.,  $\theta$  in our model). One highly appealing option is the use of conjoint analysis. In conjoint analysis,

survey respondents are presented with options that simultaneously vary two or more attributes of a product/service and are asked to indicate their preferences among these options. Respondents' preference orderings are then used to estimate the utility weights. We refer the interested reader to [5] for a detailed survey.

We assume each user is a rational decision maker in that she chooses technology  $i$  only if it provides a surplus that is both positive (Individual Rationality constraint) and higher than that of the other technology (Incentive Compatibility constraint). In other words, a user chooses

$$\begin{cases} \text{no technology} & \text{if } U_i < 0 \text{ for all } i, \\ \text{technology 1} & \text{if } U_1 > 0 \text{ and } U_1 > U_2, \\ \text{technology 2} & \text{if } U_2 > 0 \text{ and } U_2 > U_1. \end{cases}$$

To determine the number of adopters of the two technologies, we characterize *indifference points* that identify user preference values associated with changes in their decision to adopt either technology. Specifically,  $\theta_i^0(\underline{x})$ ,  $i \in \{1, 2\}$  denote thresholds where users switch from deriving negative surplus to deriving positive surplus from technology  $i$ , i.e., for a given  $\underline{x}$ ,  $\theta_i^0(\underline{x})$  is a point such that  $U_i(\theta_i^0, \underline{x}) = 0$ . Similarly,  $\theta_2^1(\underline{x})$  corresponds to the threshold where users switch from preferring technology 1 to preferring technology 2, i.e.,  $\theta_2^1(\underline{x})$  satisfies  $U_1(\theta_2^1, \underline{x}) = U_2(\theta_2^1, \underline{x})$ . By definition, any user with preference value  $\theta > \theta_i^0$  derives positive surplus from technology  $i$ . And a user with  $\theta > \theta_2^1$  derives greater surplus from technology 2 than technology 1 (assuming  $q_2 > q_1$ ).

Given these definitions, consider the following two subsets of users defined in  $[0, 1]$ ,

$$\begin{aligned} \Theta_1(\underline{x}) &= \{\theta \in [0, 1] \mid U_1(\theta, \underline{x}) \geq U_2(\theta, \underline{x}), U_1(\theta, \underline{x}) > 0\}, \\ \Theta_2(\underline{x}) &= \{\theta \in [0, 1] \mid U_2(\theta, \underline{x}) > U_1(\theta, \underline{x}), U_2(\theta, \underline{x}) > 0\}. \end{aligned}$$

Under the assumption that the functions  $U_i(\theta_i, \underline{x})$ ,  $i = 1, 2$ , are continuous, both subsets are easily shown to be connected intervals, so that if we denote as  $H_i(\underline{x})$ ,  $i = 1, 2$ , the number of users in each subset, and further assume that  $F(\theta)$  is continuous, we have the following expression for  $H_i(\underline{x})$

$$H_i(\underline{x}) = F(b_i) - F(a_i)$$

where  $[a_i, b_i] = \text{closure of } \Theta_i(\underline{x})$ .

Equilibrium adoption levels can then be characterized by:

$$x_i^* = H_i(\underline{x}^*) \text{ for } i = 1, 2. \quad (2)$$

In other words, at equilibrium, the number of users for whom it is individually rational and incentive compatible to choose technology  $i$  equals the current number of adopters of technology  $i$ . Based on this formulation, our goal is to characterize, as a function of the exogenous system parameters  $p_i, q_i$ ,  $i = 1, 2$ , the equilibrium adoption levels, i.e., the fixed points of Eqn. (2), and the dynamics leading to them.

### 3.2 Diffusion dynamics

Characterizing the dynamics of technology adoption towards equilibrium, calls for defining the decision making process of users. Suppose that at time ' $t$ ', the "current" technology adoption values,  $x_i(t)$ ,  $i = 1, 2$ , are announced to all users. Assuming this information is available to the entire population, each user could individually evaluate her utility for the two technologies and immediately make the corresponding adoption decision. Under these assumptions,  $(H_i(\underline{x}(t)) - x_i(t))$  would be the proportion of users that proceed to adopt(disadopt) technology  $i$  at  $t$ . Users' adoption

decisions may, however, be delayed for a number of reasons; e.g., not all users may learn the current adoption levels at the same time and they may react to this information at different rates. In order to capture these effects, the diffusion rate of user decisions at time  $t$  can be modeled by

$$\frac{dx_i(t)}{dt} = (H_i(\underline{x}(t)) - x_i(t))P(t) \quad (3)$$

where  $P(t)$  is the expected conditional probability that an individual who has not yet adopted technology  $i$  will do so at time  $t$ .  $P(t)$  is analogous to the hazard rate in individual-level diffusion models (e.g., see [6]). We assume that the propensity of individuals to adopt does not change with time, i.e.,  $P(t) = \gamma$ . With this assumption, the diffusion rate at time  $t$  is given by:

$$\frac{dx_i(t)}{dt} = \gamma(H_i(\underline{x}(t)) - x_i(t)), \quad i \in \{1, 2\}. \quad (4)$$

One aspect of our model needs further clarification. Specifically, the model identifies the rate of technology adoption, but not *which* users are making the change. To preserve consistency with user preferences, we assume that in our target population the users that are the first to adopt (or disadopt) technology  $i$  are those that stand to benefit the most from it. This ensures that at all times the sets of users having adopted either technology are consistent, and correspond to blocks of users with contiguous preferences.

## 4. CHARACTERIZING EQUILIBRIA

As outlined in the previous section, the dynamics of technology adoption in our model involve both identifying possible equilibria and characterizing how they are reached. We start by exploring the first aspect as specified through Eqn. (2), and for that purpose introduce the following assumptions:

- $\theta$  has a uniform distribution over  $[0, 1]$ . This assumption is for tractability and is common in the literature. Although it affects the magnitude of the equilibrium, it does not qualitatively affect the results [2].
- $v(x_i)$  is linear in  $x_i$ , i.e.,  $v(x_i) = x_i$ . This is consistent with Metcalfe's law and commonly used in the literature (e.g., [4]). Note that the maximum network benefit is normalized to 1 and all other benefits and costs are expressed in the same unit.
- The entrant technology is of higher quality, i.e.,  $q_2 > q_1$ . In other words, the technical superiority of technology 2 affords its users greater intrinsic benefits.
- $p_i > 0$ ,  $q_i > 0$ ,  $i = 1, 2$ .

### 4.1 Identification of $H_i(\underline{x})$

Recall from Eqn. (2) that equilibria are determined by the functions  $H_i(\underline{x})$ ,  $i = 1, 2$ , which themselves depend on the values of the indifference points identifying the preference levels at which users derive positive utility from either technology ( $\theta_i^0(\underline{x})$ ,  $i = 1, 2$ ) and prefer technology 2 over technology 1 ( $\theta_2^1(\underline{x})$ ) – recall our assumption that  $q_2 > q_1$ . The corresponding conditions are given by

$$U_i(\theta, \underline{x}) > 0 \quad \text{if } \theta > \theta_i^0(\underline{x}) \quad (5)$$

$$U_2(\theta, \underline{x}) > U_1(\theta, \underline{x}) \quad \text{if } \theta > \theta_2^1(\underline{x}). \quad (6)$$

**Table 1: Partitions characterizing  $H_i(x)$** 

$\theta_1^0 \geq \theta_2^0$		$\theta_1^0 < \theta_2^0$	
Region	condition	Region	condition
$R_1$	$\theta_2^0 \leq 0$	$R_4$	$\theta_2^1 \leq 0, \quad 0 \leq \theta_1^0$
$R_2$	$0 < \theta_2^0 < 1$	$R_5$	$0 < \theta_2^1 < 1, \quad \theta_1^0 \leq 0$
$R_3$	$1 \leq \theta_2^0$	$R_6$	$0 < \theta_2^1 < 1, \quad 0 < \theta_1^0 < 1$
		$R_7$	$1 \leq \theta_2^1, \quad \theta_1^0 \leq 0$
		$R_8$	$1 \leq \theta_2^1, \quad 0 < \theta_1^0 < 1$
		$R_9$	$1 \leq \theta_2^1, \quad 1 \leq \theta_1^0$

Eqn. (5) indicates that any user with  $\theta > \theta_i^0$  gets positive surplus from adopting technology  $i$ , and Eqn. (6) implies that she prefers technology 2 over technology 1 if  $\theta > \theta_2^1$  and prefers technology 1 otherwise.

Setting  $U_i(\theta, \underline{x}) = 0$ , we get

$$\theta_1^0(\underline{x}) = \frac{p_1 - x_1}{q_1} \quad (7)$$

$$\theta_2^0(\underline{x}) = \frac{p_2 - x_2}{q_2} \quad (8)$$

Similarly, setting  $U_1(\theta, \underline{x}) = U_2(\theta, \underline{x})$  gives

$$\theta_2^1(\underline{x}) = \frac{(x_1 - x_2) + (p_2 - p_1)}{(q_2 - q_1)}. \quad (9)$$

To simplify notation, we use from now on  $\theta_i^0$  and  $\theta_2^1$  instead of  $\theta_i^0(\underline{x})$  and  $\theta_2^1(\underline{x})$ . After simple manipulations, we get

$$\begin{aligned} \theta_2^1 - \theta_1^0 &= \frac{q_2}{q_2 - q_1} (\theta_2^0 - \theta_1^0), \\ \theta_2^1 - \theta_2^0 &= \frac{q_1}{q_2 - q_1} (\theta_2^0 - \theta_1^0) \end{aligned}$$

and have the following result about the location of  $\theta_1^1$ .  
**LEMMA 4.1.** *If  $\theta_1^0 < \theta_2^0$ , then  $\theta_2^1 > \theta_2^0 > \theta_1^0$ . If  $\theta_1^0 \geq \theta_2^0$ , then  $\theta_2^1 \leq \theta_2^0 \leq \theta_1^0$ .*  
 Using the above lemma, we obtain expressions for  $H_i(\underline{x})$ :

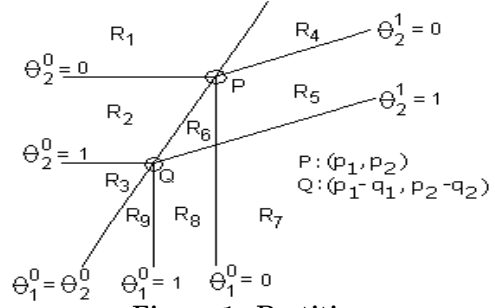
$$H_1(\underline{x}) = \begin{cases} [\theta_2^1]_{[0,1]} - [\theta_1^0]_{[0,1]} & \text{if } \theta_1^0 < \theta_2^0 \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

$$H_2(\underline{x}) = \begin{cases} 1 - [\theta_2^1]_{[0,1]} & \text{if } \theta_1^0 < \theta_2^0 \\ 1 - [\theta_2^0]_{[0,1]} & \text{otherwise} \end{cases}$$

where  $x_{[a,b]}$  is the projection of  $x$  into the interval  $[a, b]$ , i.e., is equal to  $x$  inside and either  $a$  or  $b$  outside. As the preference levels  $\theta$  of all users lie in  $[0, 1]$ , Eqn. (10), fully determine  $H_i(\underline{x})$ , albeit with possibly different expressions based on the outcome of the projections. Hence, our next step is to partition the  $(x_1, x_2)$  plane into regions where  $H_i(\underline{x})$  has a unique expression. Although somewhat tedious, this can be readily achieved from combining Eqns. (7) to (9) with Eqn. (10). The resulting partitioning is illustrated in Figure 1 that identifies nine distinct regions. These regions are further characterized in Table 1 in terms on their boundaries expressed as conditions on the indifference points. Armed with explicit, albeit multiple (one per region) expressions for  $H_i(\underline{x})$ , we can solve Eqn. (2) and identify equilibria.

## 4.2 Computing equilibria

The computation of equilibria proceeds in two steps. The first involves identifying *candidate* equilibria from solving Eqn. (2) in each of the nine regions where it takes potentially different expressions. The second consists of validating


**Figure 1: Partitions**
**Table 3: Conditions for a valid equilibrium**

candidate	conditions
(0,0)	$p_1 - q_1 > 0$ and $p_2 - q_2 > 0$
(1,0)	$p_1 \leq 1$ and $(q_2 - p_2) - (q_1 - p_1) \leq 1$
(0,1)	$p_2 \leq 1$ and $0 \leq p_1 - p_2 \frac{q_1}{q_2} + \frac{q_1}{q_2}$
$(\frac{p_1 - q_1}{1 - q_1}, 0)$	$\max\{0, p_1 - q_1\} < \frac{p_1 - q_1}{1 - q_1} < \min\{p_1, 1\}$ $q_2 - q_1 - (p_2 - p_1) < \frac{p_1 - q_1}{1 - q_1}$
$(0, \frac{p_2 - q_2}{1 - q_2})$	$\max\{0, p_2 - q_2\} < \frac{p_2 - q_2}{1 - q_2} < \min\{p_2, 1\}$ $\frac{1}{q_2} (\frac{p_2 - q_2}{1 - q_2}) \geq \frac{p_2}{q_2} - \frac{p_1}{q_1}$
$\underline{x}_{R_5}^*$	$\max\{p_1, \frac{1 - (p_2 - p_1)}{2}\} < x_{1R_5}^*$ $x_{1R_5}^* < \min\{1, \frac{1 - (p_2 - p_1) + q_2 - q_1}{2}\}$
$\underline{x}_{R_6}^*$	$0 < x_{1R_6}^* \leq \min\{p_1, 1\}, \quad 0 < x_{2R_6}^*$

these candidate equilibria by determining if they indeed belong to the region associated with the expression of  $H_i(\underline{x})$  used to compute them. The results of the first phase of this computation are reported in Table 2 for each region, together with the corresponding expressions for  $H_i(\underline{x})$ . Note the simple expressions for  $H_i(\underline{x})$ , which imply that the differential equation of Eqn. (4) can be readily solved to characterize the dynamics of technology adoption. For ease of notation in Table 2, we denote the candidate equilibrium in  $R_5$  by  $\underline{x}_{R_5}^* = (x_{1R_5}^*, 1 - x_{1R_5}^*)$  and the candidate in  $R_6$  by  $\underline{x}_{R_6}^* = (x_{1R_6}^*, x_{2R_6}^*)$ . The specific values are:

$$x_{1R_5}^* = \frac{p_2 - p_1 - 1}{q_2 - q_1 - 2} \quad (11)$$

$$x_{1R_6}^* = \frac{\frac{q_2}{q_1} p_1 - p_2 + 1 - \frac{p_1}{q_1}}{q_1 + 1 + \frac{q_2}{q_1} - q_2 - \frac{1}{q_1}} \quad (12)$$

$$x_{2R_6}^* = 1 - \frac{p_1}{q_1} + \left(\frac{1}{q_1} - 1\right) x_{1R_6}^*. \quad (13)$$

The second step of the computation, validating the equilibria listed in Table 2 by verifying that they belong to their respective region, can be carried out using again Eqns. (7) to (9) together with the conditions specified in Table 1. The resulting list of valid equilibria together with the conditions required for these validations are provided in Table 3. To illustrate how these conditions can be derived, we briefly sketch the process for  $\underline{x}_{R_5}^*$ . For  $\underline{x}_{R_5}^*$  to be an equilibrium, it should satisfy

$$C1: \quad x_1 + (p_2 - p_1) - (q_2 - q_1) < x_2 < x_1 + (p_2 - p_1)$$

$$C2: \quad x_1 \geq p_1.$$

Using the expression for  $x_{1R_5}^*$  given in Eqn. (11) in conditions C1 and C2 yields the conditions given in Table 3.

Table 2: Candidate equilibria

Region	$H_1(\underline{x})$	$H_2(\underline{x})$	Candidate $\underline{x}^*$
$R_1$	$H_1(\underline{x}) = 0$	$H_2(\underline{x}) = 1$	$(0, 1)$
$R_2$		$H_2(\underline{x}) = 1 - \frac{p_2 - x_2}{q_2}$	$(0, \frac{p_2 - q_2}{1 - q_2})$
$R_3$		$H_2(\underline{x}) = 0$	$(0, 0)$
$R_4$	$H_1(\underline{x}) = \frac{p_2 - p_1 - (x_2 - x_1)}{q_2 - q_1}$	$H_2(\underline{x}) = 1$	$(0, 1)$
$R_5$		$H_2(\underline{x}) = 1 - \frac{p_2 - p_1 - (x_2 - x_1)}{q_2 - q_1}$	$(x_{1R_5}^*, 1 - x_{1R_5}^*)$
$R_6$		$H_2(\underline{x}) = 1 - \frac{p_2 - p_1 - (x_2 - x_1)}{q_2 - q_1}$	$(x_{1R_6}^*, x_{2R_6}^*)$
$R_7$	$H_1(\underline{x}) = 1 - \frac{p_1 - x_1}{q_1}$	$H_2(\underline{x}) = 0$	$(1, 0)$
$R_8$			$(\frac{p_1 - q_1}{1 - q_1}, 0)$
$R_9$			$(0, 0)$

Table 3 not only identifies valid equilibria, but it also indicates that there exist conditions under which both technologies can coexist in equilibrium. It is interesting to note that this is realized without the presence of converters. In contrast, in a similar scenario without converters, [7] finds that only one technology exists in equilibrium and that there exists a tipping point where one technology takes over the market. This is because their model assumes a homogeneous population of users. Heterogeneity in the user population adds complexity but highlights a finding of potential interest to a policy maker, namely, that there are many cases where both technologies can survive. When cast in the context of network technologies, this can mean significant inefficiencies if it calls for introducing and maintaining two versions of new and existing service, e.g., an IPv4 and an IPv6 version. Clearly, the availability of gateways, something we plan to incorporate next, will affect this conclusion. However, cumbersome or expensive gateways, as is currently the case between IPv4 and IPv6, should introduce little changes, and as shown in [7], the availability of gateways can both hasten or slow-down the deployment of a new technology.

It should be noted that while Table 3 identifies valid equilibria, it does not specify which one is selected for combinations of parameters that satisfy more than one of the validity conditions specified in the table. Exploring this aspect together with the extent to which different outcomes can be realized through small changes in configuration parameters is a topic we explore further in the next section.

## 5. TECHNOLOGY ADOPTION DYNAMICS

This section investigates several scenarios that illustrate the broad range of dynamics and outcomes that the model allows us to elucidate. Throughout the section, technology 1 is the incumbent and technology 2 the entrant, which as we recall is assumed to be of higher quality, i.e.,  $q_2 > q_1$ .

Because of the wide range of parameter combinations that can be considered, we focus our investigation on scenarios of most practical interest. Specifically,

- $0 < x_1(0^-) < x_{1only}^*$  and  $x_2(0^-) = 0$ , where  $x_{1only}^*$  is a “stable” equilibrium if only technology 1 exists in the market. In other words, technology 1 alone would survive but has not yet achieved its equilibrium penetration at  $t = 0$ , which is when technology 2 is introduced with a penetration level of 0.
- When technology 2 is introduced at time  $t = 0$ , it is priced so that it can start gaining market share. In other words,  $\theta_2^0(x_1(0^-), 0) < 1$  and  $\theta_2^1(x_1(0^-), 0) < 1$ ,

which then implies  $p_2 < q_2$  and  $p_2 - p_1 - (q_2 - q_1) < -x_1(0^-)$ , respectively.

Note that from Table 3,  $p_2 < q_2$  implies that  $(0, 0)$  is not a valid equilibrium. Hence, our choice of  $p_i$  and  $q_i$  ensures that at least one of the two technologies survives at equilibrium.

Our first investigation highlights sensitivity to pricing of technology 2. Specifically, as illustrated in Figure 2, we see that for the same values of  $p_1 = 1.01$ ,  $q_1 = 2.95$  and  $q_2 = 5.5$ , minor differences in  $p_2$  result in drastically different outcomes that range from either technology being the sole survivor (left and right-most curves), to both technology coexisting in equilibrium (center curve). Of particular interest is the fact that all three scenarios appear qualitatively very similar, i.e., with both technologies experiencing healthy growth after the introduction of technology 2. These qualitative similarities notwithstanding, differences in growth rates between the two technologies are responsible for the vastly different end-results.

The existence of a pricing combination that allows both technologies to survive is also of interest, as is the fact that price alone, and not the initial penetration of technology 1, determines the outcome in these examples. Finally, it is also worth pointing out that in the scenario where technology 2 eventually eliminates technology 1, this result is realized at the cost of a lower overall market penetration than feasible with technology 1 alone. In other words, the higher quality but higher price technology failed to attract some of the “low-valuation” users. Further reduction in technology 2’s price can help increase its penetration among the low-valuation users. This may or may not be profitable.

Our second example, shown in Figure 3, identifies a scenario ( $p_1 = 1.2$ ,  $q_1 = 2.95$ ,  $q_2 = 5.1$  and  $p_2 = 2.55$ ), where unlike Figure 2, the final equilibrium depends on the penetration of technology 1 when technology 2 is introduced. This highlights that when technology 2 is marginally more attractive than technology 1, network externalities can be dominant in determining the outcome. Thus, the timing of introduction of technology 2 can be crucial in deciding its eventual success. As in Figure 2, both scenarios start with qualitatively similar behaviors, which makes it that much harder for the policy maker to predict the evolution of technology adoption, unless careful attention is paid to the relative growth rates of each technology. An additional point of interest in this configuration is that it includes three equilibria,  $(0.4912, 0.2686)$ ,  $(0.8974, 0)$  and  $(0, 0.6220)$ , but the one that involves both technologies coexisting is unstable. As a result, only one survives in any stable equilibrium.

Figure 4 presents our last scenario ( $p_1 = 0.5$ ,  $p_2 = 5.2$ ,  $q_1 =$

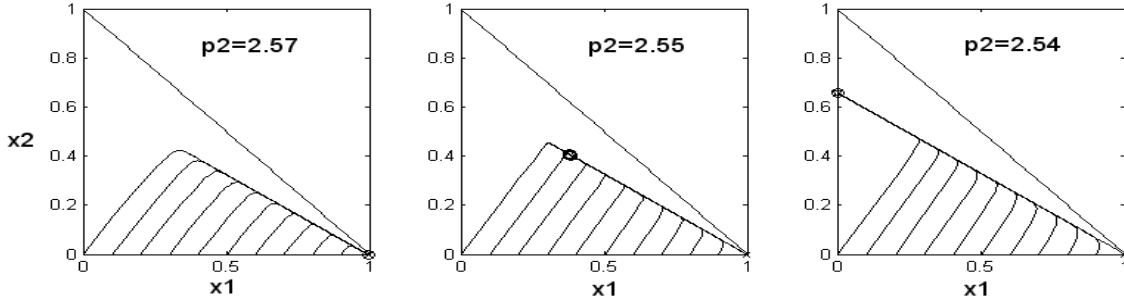


Figure 2: Effects of price changes on diffusion dynamics

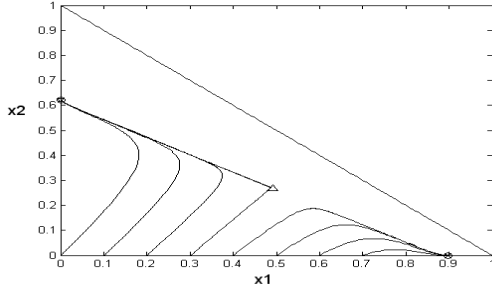


Figure 3: On the impact of initial penetration

0.3 and  $q_2 = 9.6$ ), which illustrates yet another possible behavior for technology adoption. The figure shows that depending on the initial penetration of technology 1, the equilibrium sees either the survival of only technology 2, albeit at a relatively low penetration (around 0.5), or the coexistence of both technologies with a combined penetration of a 100%. Both the differences in eventual penetration and the presence of a sharp demarcation line between the two regions are of interest. This is an instance, where the sufficient initial penetration of technology 1 ensures not only its survival, but preserves its ability to continue serving the needs of the “low end” of the market, which the better but pricier technology 2 cannot satisfy.

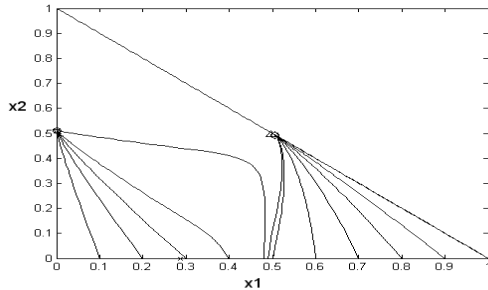


Figure 4: Coexistence vs. market penetration

## 6. CONCLUSION

The model developed in this paper is a first step towards better understanding what affects the outcome of competition between an incumbent network technology and an entrant. It accounts for network externalities and heterogeneity in how individual users make technology adoption

decisions, and captures both adoption dynamics and their eventual outcome. In spite of the model’s relative simplicity, it revealed various issues of potential interest in assessing the likelihood of success of a new networking technology

There are many directions in which this initial work can and should be extended, and we mention a few. A first is to include gateways or converters that offer some level of compatibility between the two technologies. Another less obvious extension involves developing pricing strategies. These could be static and seek to identify the optimal timing and pricing of the new technology to maximize profit<sup>1</sup>. Even more interesting are dynamic pricing strategies to maximize the odds of survival or profit of each technology.

## 7. ACKNOWLEDGMENT

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<sup>1</sup>Note that this calls for introducing a cost and revenue model for each producer of technology.